Twitch = Twee + Stitch (learning abstractions for equational theorem proving)

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DEMO

The idea

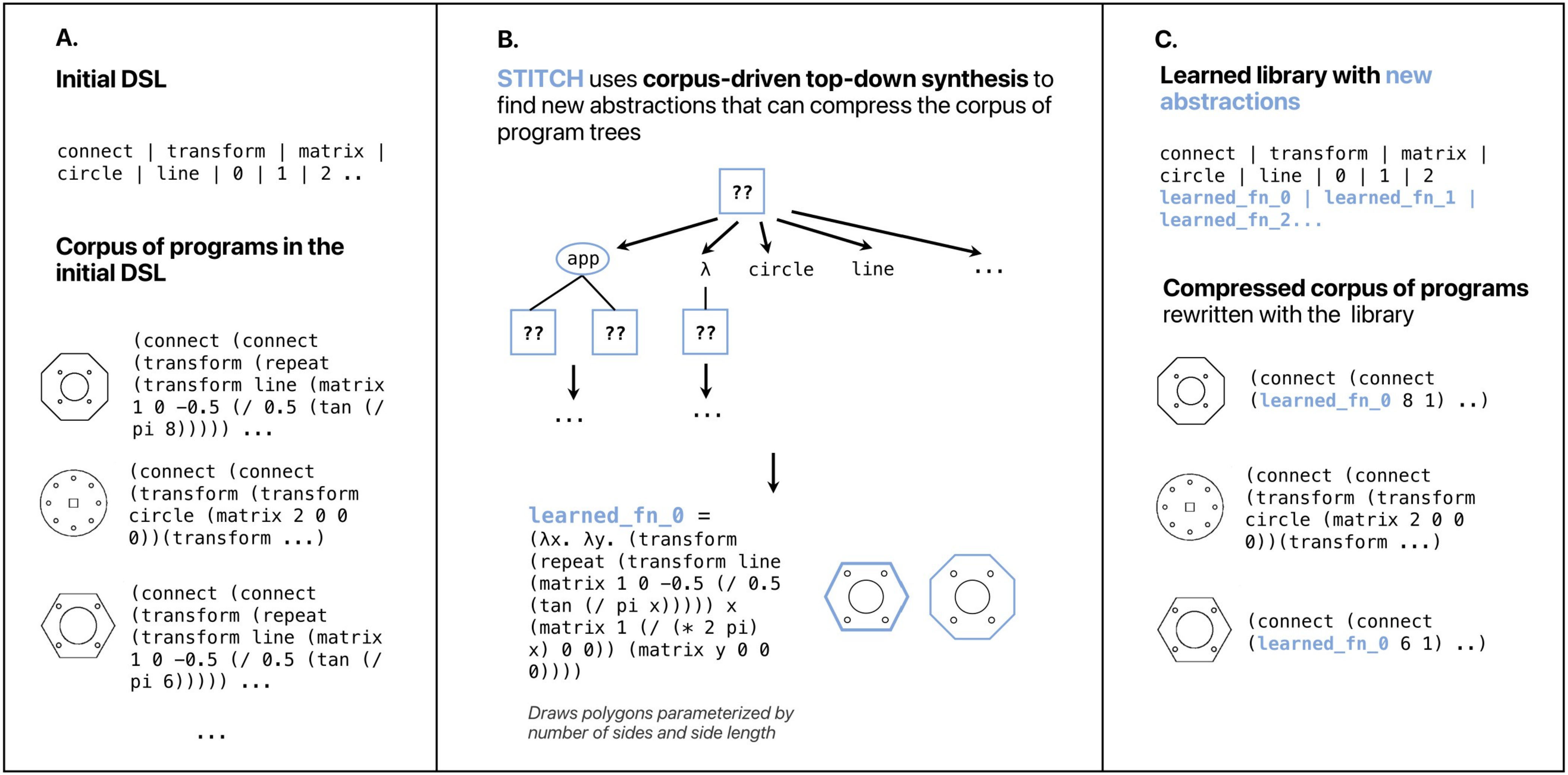
1. Automatically discover interesting term shapes

2. Give names to them

How?

How?

Stitch (“Top-down synthesis for library learning”, Bowers et al. 2023)



Stitch

f(x, f(y, f(f(z, f(x, x)), f(z, f(x, x)))))
f(g(a), g(a))

A(x) := f(x, x)

f(x, A(f(z, A(x))))
f(A(g(a)))

Learning domain abstractions

Easy: in a ring,
$x^3 = x \Rightarrow$ ring is commutative

proof! → abstractions

Medium: in a ring,
$x^4 = x \Rightarrow$ ring is commutative

proof! → abstractions

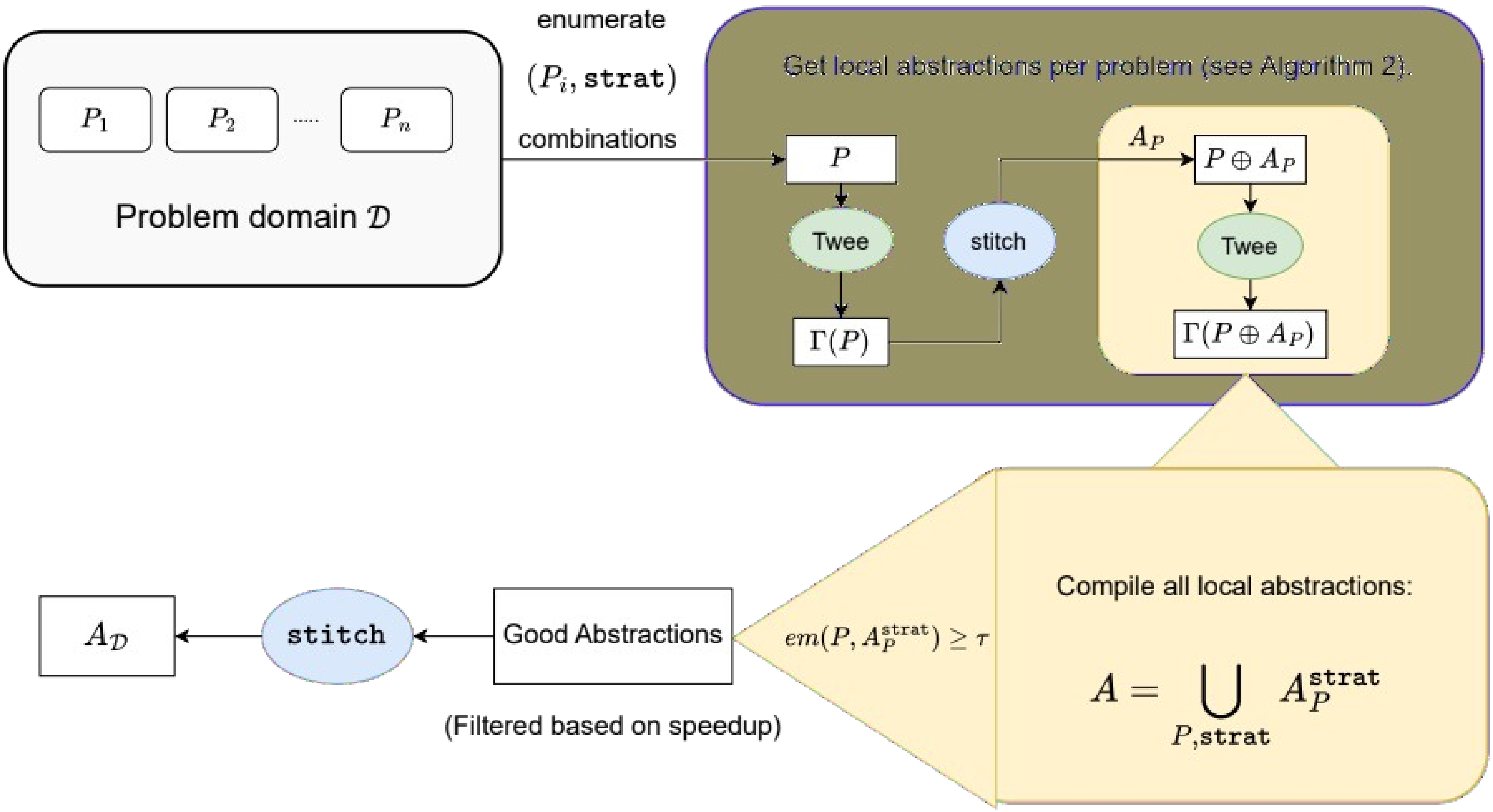
Open: in a ring,
$x^5 = x \Rightarrow$ ring is commutative

← abstractions

Medium: in a ring,
$x^6 = x \Rightarrow$ ring is commutative

proof! → abstractions

Learning domain abstractions



Hints for LAT075-1

- $f(A, A)$
- $f(f(B, A), f(B, A))$
- $f(f(A, B), f(A, C))$
- $f(f(D, C), f(B, A))$
- $f(f(f(C, f(C, f(f(A, A), B))), B), A)$

Hints for ROB001-1

add(negate(add(A, B)), negate(add(negate(A), B)))

negate(add(negate(add(A, B)), negate(add(B, C))))

negate(add(negate(add(A, B)), negate(add(A, C))))

negate(add(A, B))

negate(add(A, negate(A)))

add(A, add(B, C))

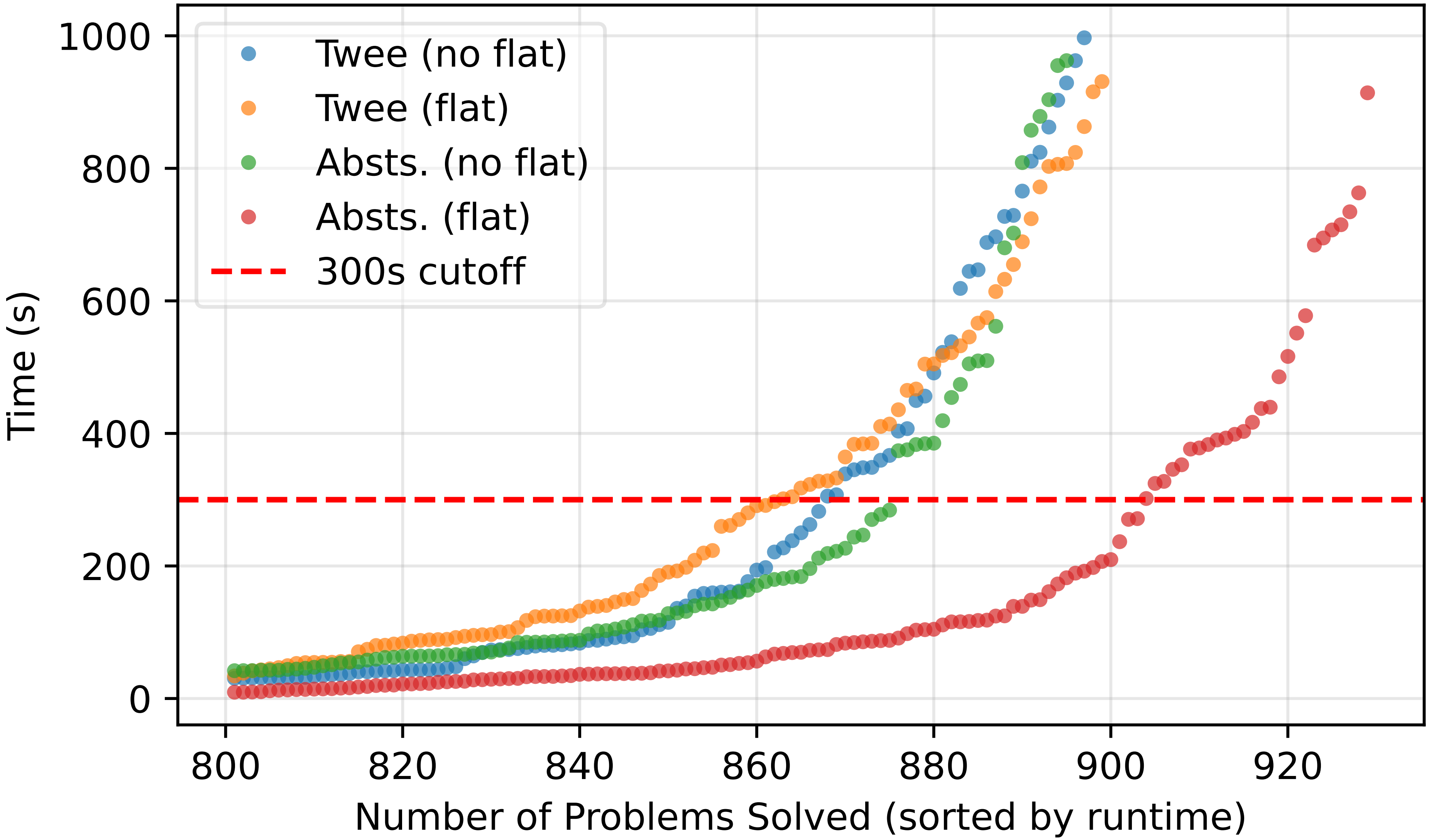
add(negate(add(A, B)), negate(add(C, D)))

negate(add(negate(add(A, B)), negate(add(A, negate(B)))))

negate(add(A, negate(B)))

Results

12 rating-1 problems



Abstractions in Twee

How to deal with abstractions?

$$(x*y)+(x*z)$$

How to deal with abstractions?

$$A(x, y, z) = (x*y)+(x*z)$$

How to deal with abstractions?

~~A(x, y, z) = (x*y)+(x*z)~~

Increases the search space a lot!

Twee architecture

Rewrite rules

$0+x \rightarrow x$

$x + (-x) \rightarrow 0$

$(x+y)+z \rightarrow x+(y+z)$

$x+(-x)+y \rightarrow 0+y$

Possible inferences (critical pairs)

$x+(-x+y) \leftarrow (x+(-x))+y \rightarrow 0+y$

…

~10,000 rules

~100,000,000 critical pairs

Scoring function picks the best one

Abstractions in the scoring function

$$c*((g(f(a), b)*f(x)) + (g(f(a), b)*g(y, z))) = g(a, b)$$

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21

Abstractions in the scoring function

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$$c*((g(f(a), b)*f(x)) + (g(f(a), b)*g(y, z))) = g(a, b)$$

21

Abstractions in the scoring function

$A(x, y, z) := (x*y)+(x*z)$

$c*((\mathbf{g(f(a), b)*f(x)}) + (\mathbf{g(f(a), b)*g(y, z)})) = g(a, b)$

21

Abstractions in the scoring function

$$A(x, y, z) := (x*y)+(x*z)$$

$$c*\mathbf{((g(f(a), b)*f(x)) + (g(f(a), b)*g(y, z)))} = g(a, b)$$

$$c*\mathbf{A(g(f(a), b), f(x), g(y, z))} = g(a, b)$$

Abstractions in the scoring function

$$A(x, y, z) := (x*y)+(x*z)$$

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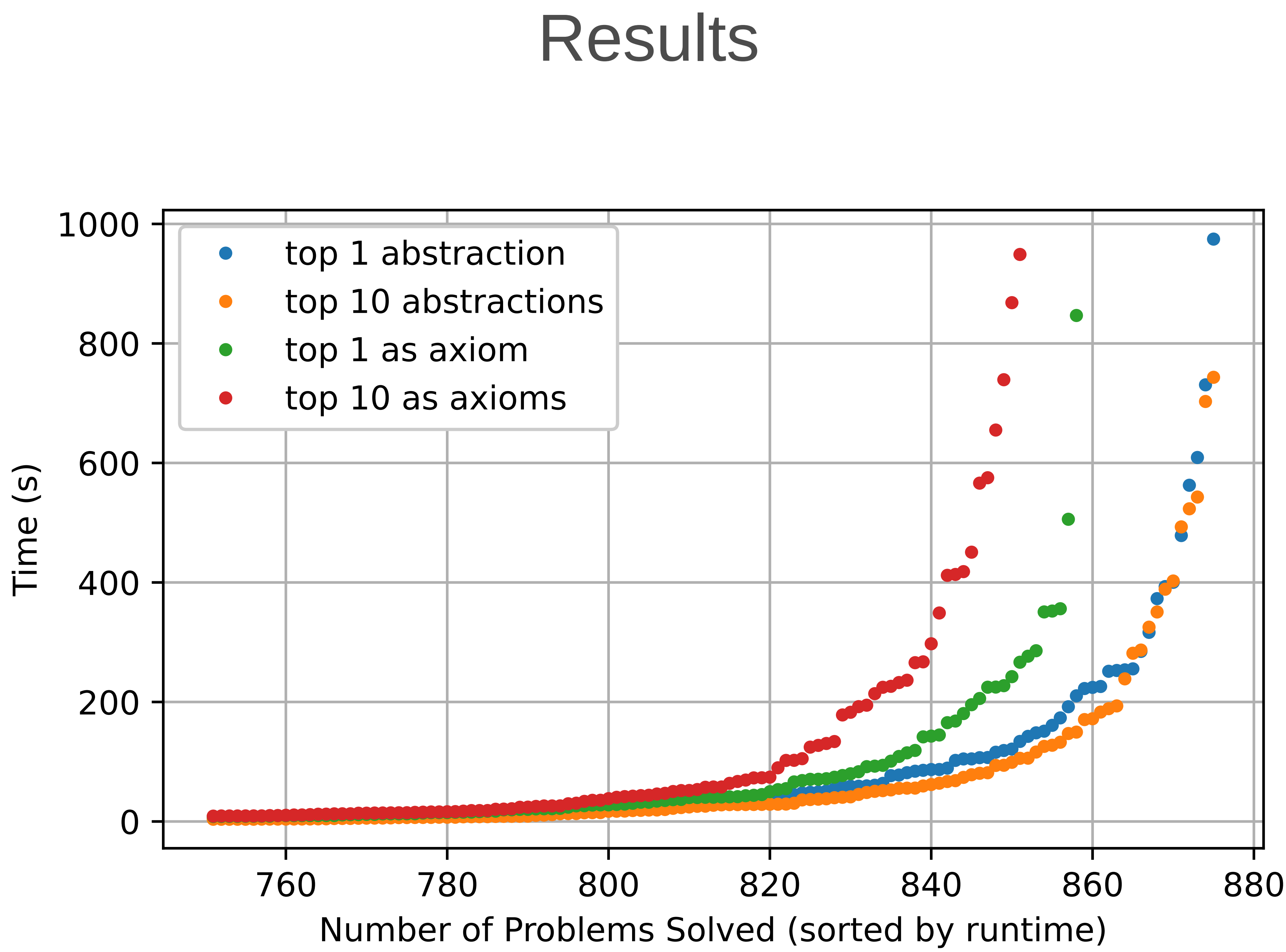
$$c*\mathbf{A(g(f(a), b), f(x), g(y, z))} = g(a, b)$$

15



Results
top 1 abstraction
top 10 abstractions
top 1 as axiom
top 10 as axioms
Time (s)
0
200
400
600
800
1000
760
780
800
820
840
860
880
Number of Problems Solved (sorted by runtime)

Limitations

$A(x, y) := f(x*y)+g(y)$

$f((a+b)*c) + g(a+b)$ ✅

$g(a+b) + f((a+b)*c)$ ❓

$\mathbf{A'(x, y) := g(y) + f(x*y)}$

Too many abstractions generated!

Conclusion

Lots of things to fine-tune…

…but promising!

Next step: try to prove a challenge problem