

**Twitch = Twee + Stitch**  
(learning abstractions for  
equational theorem proving)

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**DEMO**

## The idea

1. Automatically discover interesting term shapes

2. Give names to them



How?



How?

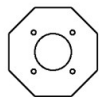
# Stitch (“Top-down synthesis for library learning”, Bowers et al. 2023)

A.

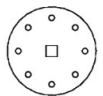
## Initial DSL

```
connect | transform | matrix |
circle | line | 0 | 1 | 2 ..
```

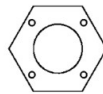
## Corpus of programs in the initial DSL



```
(connect (connect
(transform (repeat
(transform line (matrix
1 0 -0.5 (/ 0.5 (tan (/
pi 8)))))) ...
```



```
(connect (connect
(transform (transform
circle (matrix 2 0 0
0))(transform ...)
```

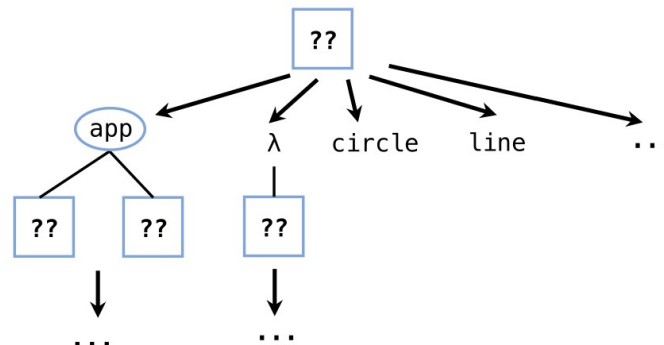


```
(connect (connect
(transform (repeat
(transform line (matrix
1 0 -0.5 (/ 0.5 (tan (/
pi 6)))))) ...
```

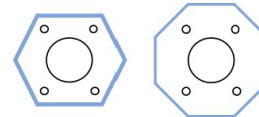
...

B.

**STITCH** uses **corpus-driven top-down synthesis** to find new abstractions that can compress the corpus of program trees



```
learned_fn_0 =
(λx. λy. (transform
(repeat (transform line
(matrix 1 0 -0.5 (/ 0.5
(tan (/ pi x)))))) x
(matrix 1 (/ (* 2 pi)
x) 0 0) (matrix y 0 0
0))))
```



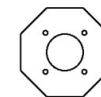
*Draws polygons parameterized by number of sides and side length*

C.

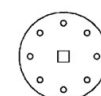
## Learned library with new abstractions

```
connect | transform | matrix |
circle | line | 0 | 1 | 2
learned_fn_0 | learned_fn_1 |
learned_fn_2...
```

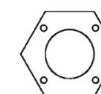
## Compressed corpus of programs rewritten with the library



```
(connect (connect
(learned_fn_0 8 1) ..)
```

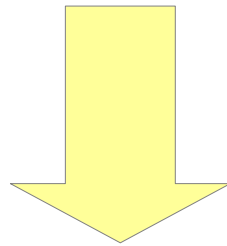


```
(connect (connect
(transform (transform
circle (matrix 2 0 0
0))(transform ...)
```



```
(connect (connect
(learned_fn_0 6 1) ..)
```

# Stitch

$$f(x, f(y, f(f(z, f(x, x)), f(z, f(x, x))))))$$
$$f(g(a), g(a))$$

$$A(x) := f(x, x)$$
$$f(x, A(f(z, A(x))))$$
$$f(A(g(a)))$$

# Learning domain abstractions

**Easy:** in a ring,  
 $x^3 = x \Rightarrow$  ring is commutative

**Medium:** in a ring,  
 $x^4 = x \Rightarrow$  ring is commutative

**Open:** in a ring,  
 $x^5 = x \Rightarrow$  ring is commutative

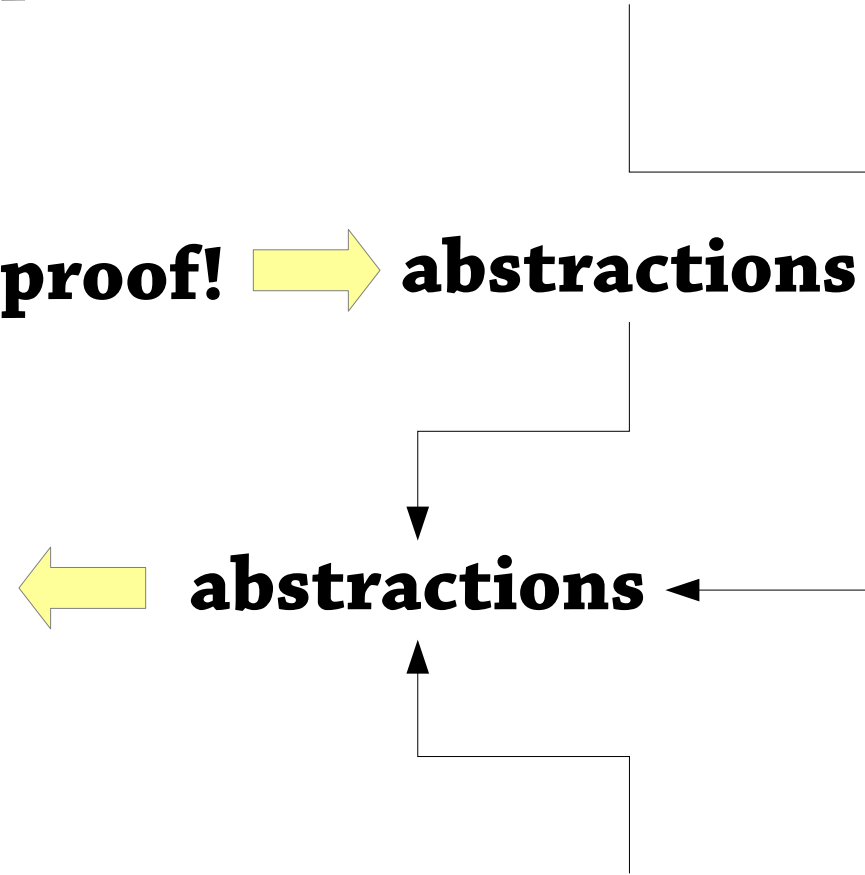
**Medium:** in a ring,  
 $x^6 = x \Rightarrow$  ring is commutative

**proof!**  **abstractions**

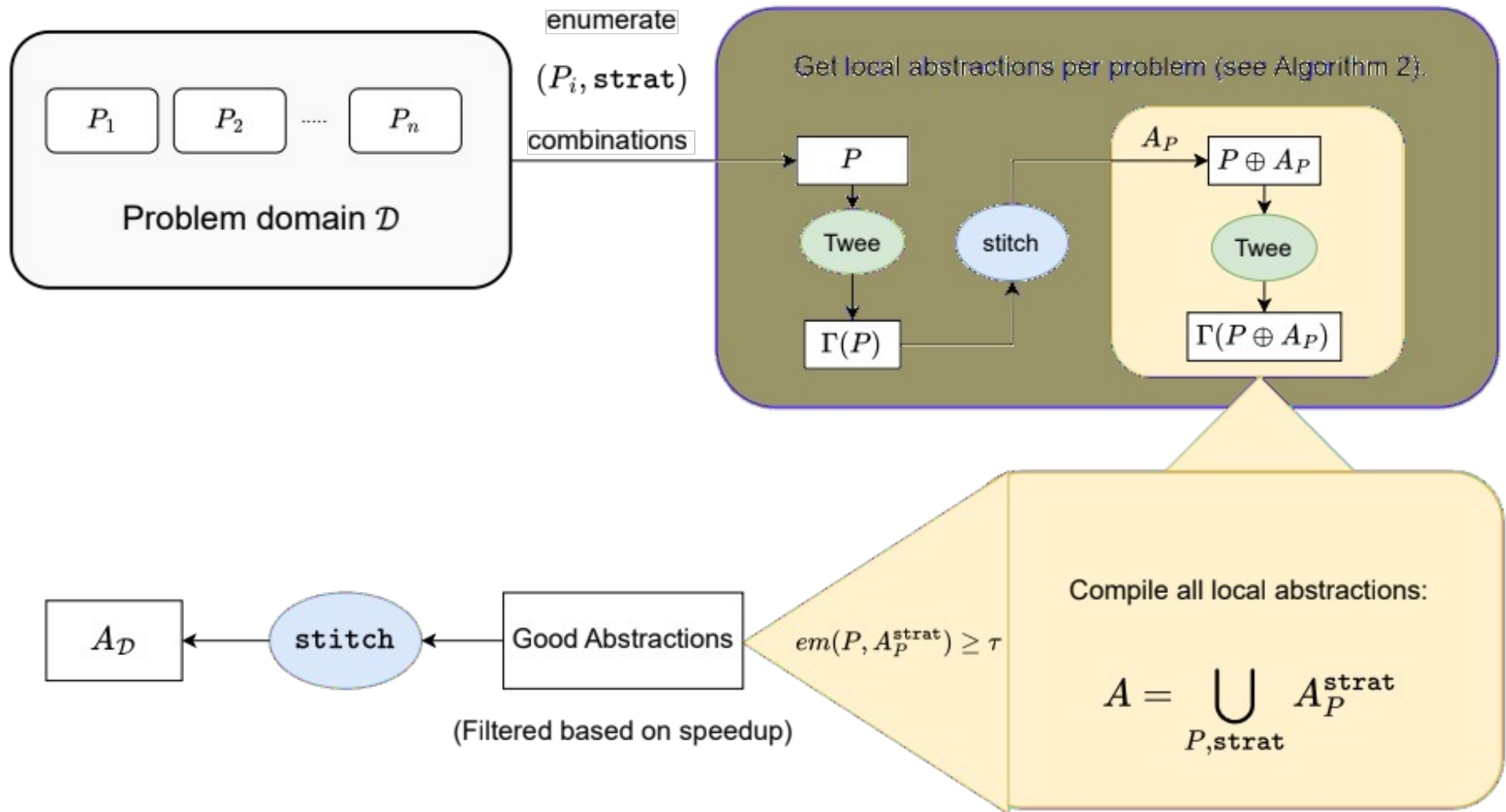
**proof!**  **abstractions**

 **abstractions** 

**proof!**  **abstractions**



# Learning domain abstractions



## Hints for LAT075-1

- $f(A, A)$
- $f(f(B, A), f(B, A))$
- $f(f(A, B), f(A, C))$
- $f(f(D, C), f(B, A))$
- $f(f(f(C, f(C, f(f(A, A), B))), B), A)$

# Hints for ROB001-1

**add(negate(add(A, B)), negate(add(negate(A), B)))**

negate(add(negate(add(A, B)), negate(add(B, C))))

negate(add(negate(add(A, B)), negate(add(A, C))))

**negate(add(A, B))**

**negate(add(A, negate(A)))**

add(A, add(B, C))

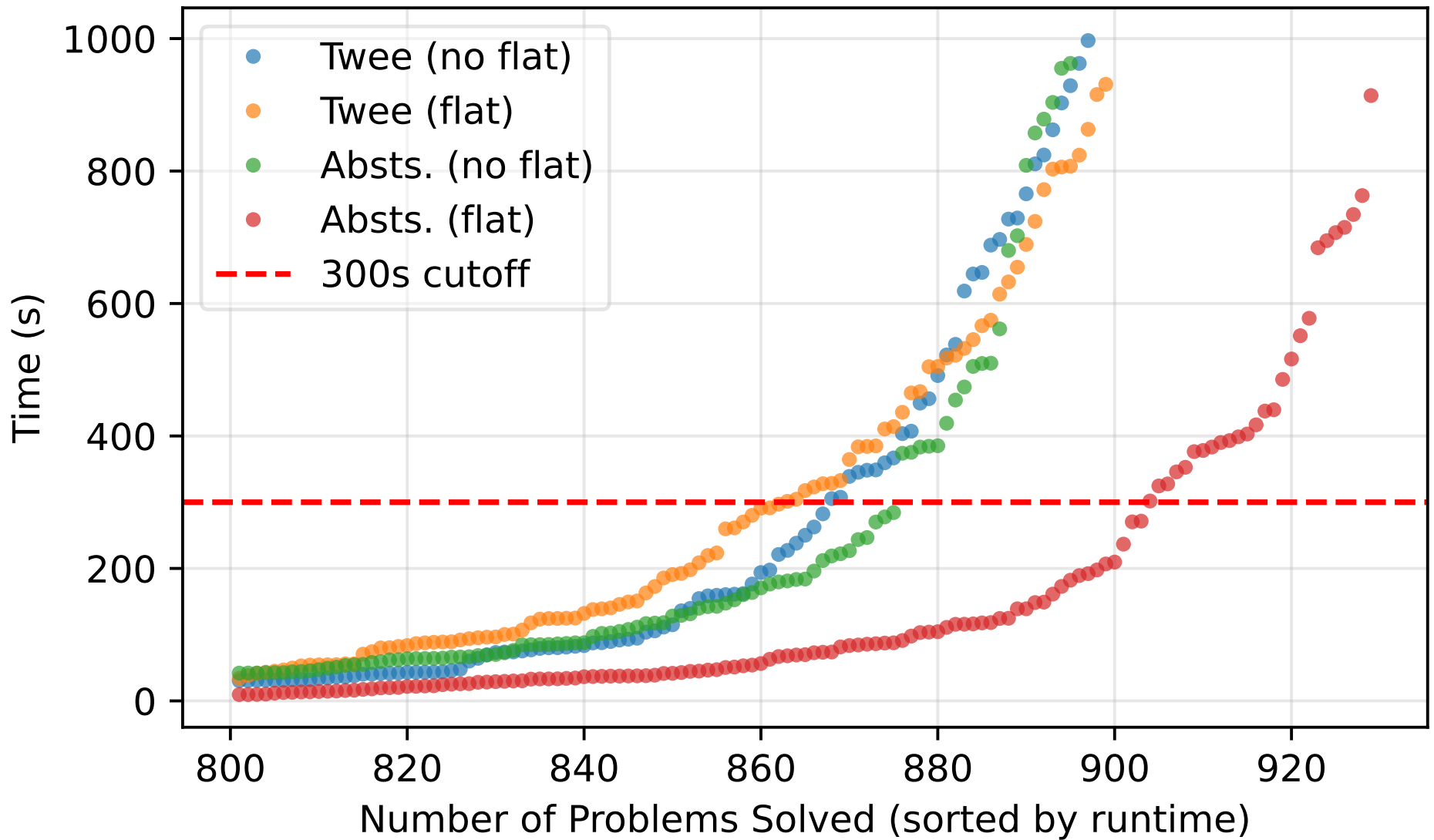
add(negate(add(A, B)), negate(add(C, D)))

negate(add(negate(add(A, B)), negate(add(A, negate(B)))))

negate(add(A, negate(B)))

# Results

12 rating-1 problems



# **Abstractions in Twee**

# How to deal with abstractions?

$$(x^*y) + (x^*z)$$

# How to deal with abstractions?

$$A(x, y, z) = (x * y) + (x * z)$$

# How to deal with abstractions?

$$\cancel{A(x, y, z) = (x*y) + (x*z)}$$

Increases the search space a lot!

# Twae architecture

## Rewrite rules

$$0 + x \rightarrow x$$

$$x + (-x) \rightarrow 0$$

$$(x + y) + z \rightarrow x + (y + z)$$

$$x + (-x) + y \rightarrow 0 + y$$

~10,000 rules

## Possible inferences

(critical pairs)

$$x + (-x + y) \leftarrow (x + (-x)) + y \rightarrow 0 + y$$

...

~100,000,000 critical pairs

Scoring function  
picks the best one

# Abstractions in the scoring function

$$c^*((g(f(a), b)^*f(x)) + (g(f(a), b)^*g(y, z))) = g(a, b)$$

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**21**

# Abstractions in the scoring function

$$A(\mathbf{x}, y, z) := (\mathbf{x} * y) + (\mathbf{x} * z)$$

$$c * ((g(f(a), b) * f(\mathbf{x})) + (g(f(a), b) * g(y, z))) = g(a, b)$$

**21**

# Abstractions in the scoring function

$$A(\mathbf{x}, y, z) := (\mathbf{x}^*y) + (\mathbf{x}^*z)$$

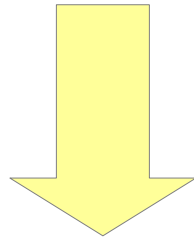
$$c^*((g(\mathbf{f}(\mathbf{a}), \mathbf{b})^*f(\mathbf{x})) + (g(\mathbf{f}(\mathbf{a}), \mathbf{b})^*g(y, z))) = g(\mathbf{a}, \mathbf{b})$$

**21**

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$$c * ((g(f(a), b) * f(x)) + (g(f(a), b) * g(y, z))) = g(a, b)$$

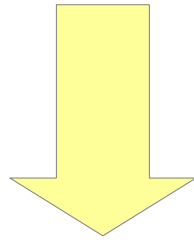


$$c * A(g(f(a), b), f(x), g(y, z)) = g(a, b)$$

# Abstractions in the scoring function

$$A(x, y, z) := (x * y) + (x * z)$$

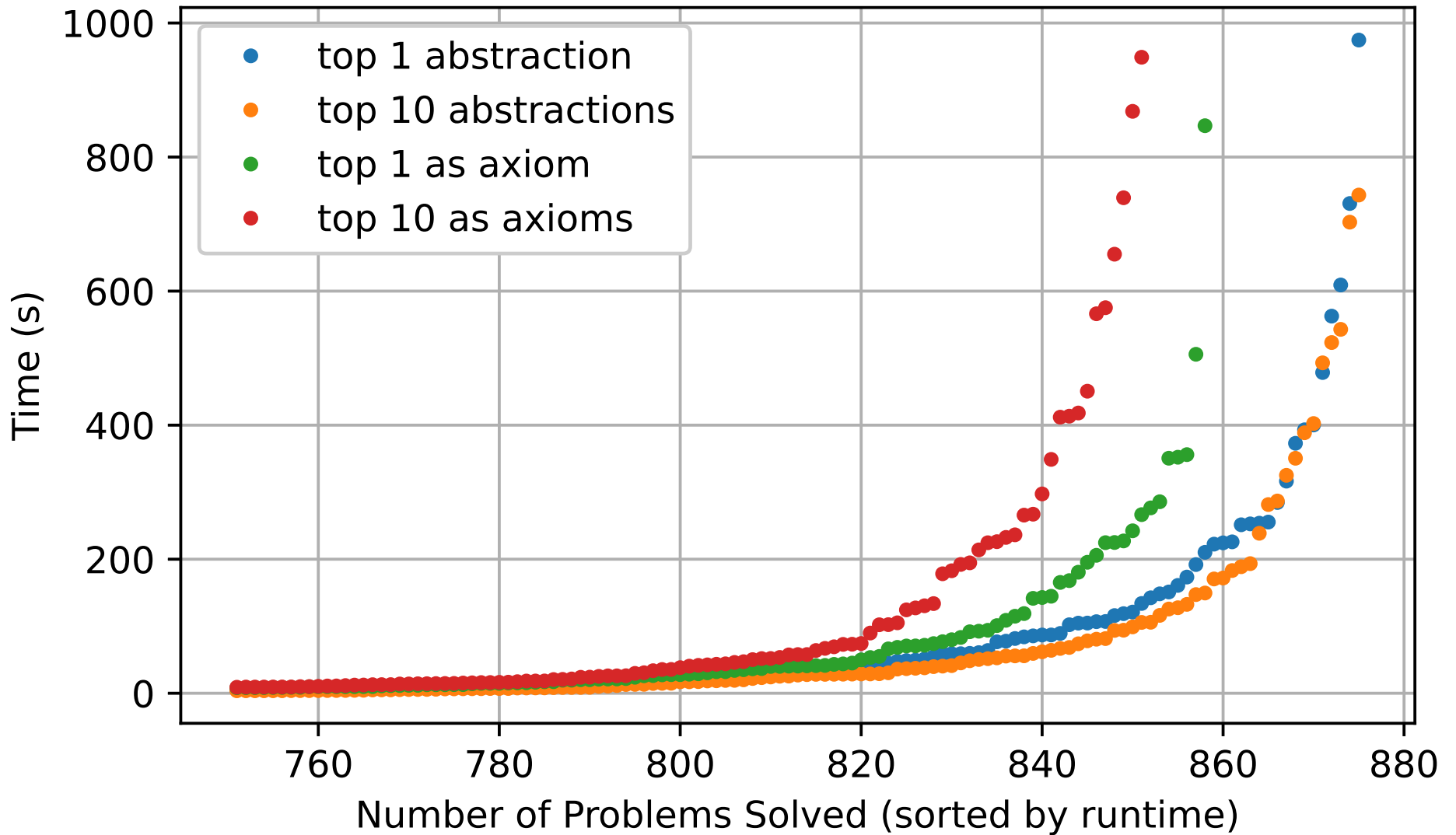
$$c * ((g(f(a), b) * f(x)) + (g(f(a), b) * g(y, z))) = g(a, b)$$



$$c * A(g(f(a), b), f(x), g(y, z)) = g(a, b)$$

**15**

# Results



# Limitations

$$A(x, y) := f(x*y) + g(y)$$

$$f((a+b)*c) + g(a+b)$$



$$g(a+b) + f((a+b)*c)$$



$$A'(x, y) := g(y) + f(x*y)$$

Too many abstractions  
generated!

# Conclusion

Lots of things to fine-tune...

...but promising!

Next step: try to prove a challenge problem